Optimal monetary policy for a small open economy

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Abstract

This paper focuses on the design of monetary policy rules for a small open economy. The model features optimizing behavior, general equilibrium and price stickiness. The real exchange rate is shown to affect the firm’s real marginal cost, aggregate supply and aggregate demand. The welfare objective depends on the openness of the economy, and the optimal policy rule differs from that which obtains in a closed economy. The inflation versus output gap stabilization trade-off is caused by the real exchange rate. The implied optimal monetary policy regime is domestic inflation target coupled with controlled floating of the real exchange rate.

1. Introduction

It is widely recognized that the exchange rate plays an important role in the transmission of the monetary policy in open economies. Recent studies on this issue include Benigno and Benigno (2000), Benigno (2004), Devereux (2004), Gali and Monacelli (2005), Taylor (2001), Svensson (2000), among others. Differently from the closed economy framework, however, where there are some stylized results concerning to an optimal policy built on interest rate rules, several questions remain object of analysis in the open economy. Specifically, there lacks a precise answer as to whether the monetary authority should respond to exchange rate movements.

The open economy environment yields additional complications to the optimal monetary policy problem. To see this, assume that the monetary authority follows an interest rate rule, as it is the common practice among central banks of developed countries, and the target variables are inflation and output gap. Exchange rate movements have a direct effect on both aggregate demand (switching demand effect) and prices (consumer price effect) and so have an indirect effect on the monetary policy instrument. On its turn, the exchange rate itself, by no arbitrage in international financial markets, is sensible to interest rate differentials. Thus, the exchange rate channels of transmission affect both private agents’ expectations of future variables and optimal response of the monetary policy to stabilize the economy.

The goal of this paper is to analyze the role of the exchange rate in the design of monetary policy rules for a small open economy. In the model economy, private agents are forward looking and the monetary policy affects economic activity through an interest rate rule. Nominal product prices are set by individual firms in a staggered manner, à la Calvo (1983). The channels of transmission imply that changes in the country’s exchange rate affect firms’ marginal cost, aggregate supply, and aggregate demand.

The model provides a theoretical background for the inflation versus output gap stabilization trade-off. It is caused by the real exchange rate. The implied policy regime is domestic inflation target, coupled with a dirty floating of the exchange rate. Because of the exchange rate channels, neither the canonical representation, proposed by Gali and Monacelli (2005), nor the isomorphic solution for the interest rate rule, suggested by Clarida, Gali, and Gertler (2001), holds in the present model. In addition, in the welfare loss, the monetary authority places a higher weight on output gap stabilization than in the closed economy counterpart, as derived by Woodford (2003). This finding suggests that an overly aggressive inflation stabilization policy might cause undesirable instability in the small open economy.
(2001) does not find stabilization trade-off under the optimal policy of complete domestic inflation stabilization. When prices are set in local currency and the law of one price does not hold, Kollmann (2002) also argues that it is optimal to stabilize domestic inflation by assuming a Taylor type of policy rule, though it implies significant exchange rate volatility. Devereux and Engel (2003) claim that prices set in producer’s currency (PCP) lead to fully flexible exchange rate. This result also does not hold here because of the terms of trade distortion and exchange rate channels of transmission, which are not considered by those authors. Contrary to the previous findings, Svensson (2000) suggests to target CPI inflation because this regime produces small to moderate variability in inflation, output and exchange rate. This result, however, is determined by the ad hoc structure of Svensson’s model, which assumes a variety loss functions. In the present model, where welfare loss is utility-based, the optimal policy is domestic inflation target.

The paper is organized as follows. The next section presents the model economy. The structural equations for aggregate demand and aggregate supply are derived in Sections 3 and 4, respectively. Section 5 defines the rational expectations equilibrium. Section 6 derives the welfare objective and the optimal monetary policy rule. Finally, Section 7 is dedicated to the concluding remarks.

2. Open economy model

2.1. Household side

The world economy is made up of two asymmetric countries, represented by a small open economy, which will be qualified later, and a foreign country (or the rest of the world economy). The small open economy is inhabited by infinitely-lived consumer-producers agents, whose total is normalized to one. There are \( j \in [0,1] \) imperfect substitute goods, each one being produced by a different producer in a monopolistic competitive basis.\(^1\) The representative agent’s utility function is specialized to be:

\[
E_t \left\{ \sum_{k=0}^{\infty} \beta^k U \left( C_{t+k} \cdot M_{t+k} \frac{P_{t+k}}{M_{t+k}} \right) - V(N_{t+k}) \right\}
\]

(1)

with \( U \left( C_{t+k} \cdot M_{t+k} \frac{P_{t+k}}{M_{t+k}} \right) = \frac{C_{t+k}^{\gamma}}{\gamma} + \frac{\alpha M_{t+k}^{\lambda}}{1-\lambda} \) and \( V(N_{t+k}) = \frac{(N_{t+k})^{\gamma}}{\gamma} \), where \( \beta \in [0,1], \sigma, \gamma_n > 0 \), and \( E_t \) is the expectations operator conditional on time \( t \) information set. The particular form of \( V(N_{t+k}) \) reflects the assumption that there is no friction in the labor market and, because wages are flexible, all workers earn the same wage and work the same number of hours.

The small open economy consumption index is a composite of home- and foreign-country-produced goods, given by:

\[
C_t = \left[ (1-\alpha) C_{H,t}^{\gamma} + \alpha C_{F,t}^{\gamma} \right]^{\frac{1}{\gamma}}
\]

(2)

where \( C_{H,t} \) is the domestic consumption of the home-produced good, \( C_{F,t} \) is the domestic consumption of the foreign-country-produced good, and \( \eta = 0 \) is the elasticity of substitution between home- and foreign-country goods. The parameter \( \alpha \in [0,1] \) measures the share of the total consumption of the foreign-country-produced goods in the home country total consumption and \( \alpha \) is referred to as the degree of openness of the small open economy.

The consumption subindexes are given by CES aggregators according to Dixit and Stiglitz (1977). Specifically, one has that:

\[
\left[ \int_{0}^{1} C_{H,t}^{\gamma} d\bar{\mu} \right]^{\frac{\gamma}{\gamma+1}} = \left[ \int_{0}^{1} C_{F,t}^{\gamma} d\bar{\mu} \right]^{\frac{\gamma}{\gamma+1}}
\]

and

\[
C_{H,t}^{\gamma} = \frac{1}{\gamma} \left[ \int_{0}^{1} C_{F,t}^{\gamma} d\bar{\mu} \right]^{\frac{\gamma}{\gamma+1}}
\]

where \( \gamma > 1 \) is the elasticity of substitution across goods produced within a country.

In a similar fashion, the home price of the home-produced good and the home price of the foreign-produced good are expressed as:

\[
P_{H,t} = \left[ \int_{0}^{1} P_{F,t} d\bar{\mu} \right]^{\frac{1}{\gamma}}
\]

and

\[
P_{F,t} = \left[ \int_{0}^{1} P_{F,t} d\bar{\mu} \right]^{\frac{1}{\gamma}}
\]

respectively. \( P_{H,t} \) is referred to as producer price index (PPI). The consumer price index (CPI) is defined equivalently to the total home consumption as:

\[
P_t = \left[ (1-\alpha) P_{H,t}^{1+\eta_1} + \alpha P_{F,t}^{1+\eta_1} \right]^{\frac{1}{\eta_1}}.
\]

Demand functions, resulting from the household’s optimal decision of allocating a fixed amount of nominal income within each category of goods, are given by:

\[
C_{H,t} = \left( \frac{P_{H,t}}{P_t} \right)^{-\gamma_1}_1 C_t \quad \text{and} \quad C_{F,t} = \left( \frac{P_{F,t}}{P_t} \right)^{-\gamma_1}_1 C_t,
\]

respectively.

The home-currency nominal budget constraint faced by the representative household can be written as:

\[
P_t G_t + D_{t+1} B_t + M_{t+1} + \zeta C_{t+1} = W_t N_t + \Pi_t + B_{t-1} + M_{t-1} + \gamma_t B_{t-1} + TR_t
\]

(3)

The notation is home country bond (\( B_t \)), money balances (\( M_t \)), total wages (\( W_t N_t \)), profits from the ownership of the firm (\( \Pi_t \)), and total lump-sum transfers from the government (\( TR_t \)). In addition, one has that \( P_t \) is the CPI, \( D_{t+1} \) is the price of the one-period domestic bond,\(^2\) and \( \zeta_t \) is the nominal exchange rate, defined as the home price of the foreign currency. As usual, an asterisk indicates a foreign country variable.

It is assumed that there is a complete set of state-contingent claims in the international financial markets. The household’s optimization problem is standard and the solution yields, after taking expectations conditional on period \( t \) information set, a stochastic consumption Euler, Eq. (4), an optimal decision rule relating consumption versus income, Eq. (5), a money demand, Eq. (6), and the household’s optimal labor supply schedule, Eq. (7).\(^3\)

\[
D_{t+1} \beta^{C_{t+1}} = \beta \left( C_{t+1} \frac{P_{t+1}}{P_t} \right)
\]

(4)

\[
D_{t+1} \alpha^{C_{t+1}} = \beta \left( C_{t+1} \frac{P_{t+1}}{P_t} \right)
\]

(5)

\[
C_{t+1} = \frac{\alpha}{\gamma} \left( \frac{M_t}{P_t} \right)^{\gamma_1} + \beta \left( C_{t+1} \frac{P_{t+1}}{P_t} \right)
\]

(6)

\[
C_{t+1} \frac{W_t}{P_t} = N_t \gamma_1
\]

(7)

There is no arbitrage in international financial markets. One can then combine consumption Euler equations to get a nominal version of the uncovered interest rate parity (UIP). In log-linear form, it can be expressed as:

\[
e^{-\gamma_t} e^{-\gamma_t} = i_t - i_t + \xi_t
\]

(8)

where \( e_t = \log(P_t) \), \( i_t = \log(1+i_t) \), and \( \xi_t = \log(1+i_t) \), and \( \xi_t \) is the risk-premia.
Eq. (8) links exchange rate and interest rate differential in the small open economy and is an additional equilibrium condition in the model. The real version of the UIP determines the real exchange rate, which is related to the terms of trade. The monetary authority has an incentive to use the monetary policy to improve the country's terms of trade and raise welfare. Corsetti and Pesenti (2001) and Corsetti et al (2000) show that monetary contraction induces real appreciation that rises the purchasing power of domestic consumers. The improved terms of trade can more than offset the resulting reduction in output and increase welfare. Deviations from the UIP are often captured by the exogenous risk-premia shock, \( \xi \), which renders the UIP an additional equilibrium condition in the model's dynamics.\(^4\)

The foreign country is assumed to have the same preferences as the small open economy. Its consumption Euler equation is:

\[
D^{ct}_{t+1} = \beta \left( C^{ct}_{t+1} \right)^{1-\alpha} P^*_t \frac{P^t_t}{P^*_t} \tag{9}
\]

Notice that the law of one price holds in the model, but the real exchange rate fluctuates over time due to differences in the composition of home and foreign country consumption baskets, for instance. Empirical evidence on the behavior of real exchange rates have shown that they are highly volatile and persistent.\(^5\) For further reference, let the log-linear version of the real exchange rate be:

\[
q_t = c_t + \tilde{P}_t \tag{10}
\]

By complete international financial markets and no-arbitrage, one can combine the Euler equations from both countries to get a risk-sharing condition. After iterating backwards, a log-linear version of the resulting equation, ignoring the irrelevant constant that depend on the initial conditions, can be written as:\(^6\)

\[
c_t = c_t^* + \frac{1}{\sigma} q_t \tag{11}
\]

Eq. (11) describes the link between relative consumption across the two countries and the real exchange rate.\(^7\) For any given real exchange rate, the consumption differential is higher the greater the elasticity of intertemporal substitution in consumption \( \tilde{C} \).

The small open economy has no effect on the steady state equilibrium conditions of the foreign country economy. This assumption implies that the foreign country can be seen as a closed economy, whose equilibrium conditions are exogenous to the small open economy.\(^8\) As a result, one has that \( c_t^* = c_t^* \) and \( \tilde{P}_t = \tilde{P}_t \).

The foreign economic policy follows a nominal interest rate rule that succeeds in stabilizing both inflation and output gap. This policy, under certain circumstances, achieves zero inflation and zero output gap in the steady state equilibrium. In order to capture the effects of foreign shocks to the small open economy, it is assumed that the foreign country nominal interest rate follows a stationary first-order autoregressive process, with \( \rho_s \in (0,1) \) and \( \nu_t \sim i.i.d. (0, \sigma^2) \).

Under the law of one price and the small open economy assumption, the CPI inflation \( (\pi_t) \) depends on PPI inflation \( (\pi_{it}) \) and changes in the real exchange rate \( (\Delta q_t) \) as follows:

\[
\pi_t = \pi_{it} + \frac{\alpha}{1-\alpha} \Delta q_t \tag{12}
\]

From the definition of the terms of trade \( (S_t = \frac{P^*_t}{P^t_t}) \) one can derive a log-linear relationship between real exchange rate and the terms of trade as:

\[
q_t = (1-\alpha) s_t \tag{13}
\]

which allows to write Eq. (11) also as a function of the terms of trade.

2.2. Production sector

The differentiated good \( j \) is produced by a firm that uses a linear technology that depends only on labor as input. The production function of firm \( j \) takes the form:

\[
Y_t(j) = A_n N_t(j) \tag{14}
\]

where \( A_n \) is a technological shock, assumed to follow a stationary first-order, log-linear autoregressive process, with \( \rho_n \in (0,1) \) and \( \nu_t \sim i.i.d. (0, \sigma^2) \).

Define \( Y_t = \left( \overline{q}_t Y_t(j) \right) \) as an index for aggregate output, analogously to the consumption index. Also, let the aggregate labor demand be defined as \( N_t = \sum_q N_t(j) q_t \). One can then show that up to a first-order approximation, the aggregate relationship \( y_t = n_t + q_t \) holds.\(^9\)

The firm’s real marginal cost, in log-linear form, is given by:

\[
m_t = w_t - \alpha_1 - \rho_n t \tag{15}
\]

Market clearing condition for good \( j \) requires:

\[
Y_t(j) = C_{H,t}^j(j) + C_{M,t}^j(j) \tag{16}
\]

Using previous equalities for \( C_{H,t}^j(j) \) and \( C_{M,t}^j(j) \) along with the corresponding versions for the foreign country, Eq. (11) before linearization, the definition of real exchange rate, a condition for a balanced steady state trade balance in the small open economy \( \left( \frac{\pi}{\sigma} = \alpha \right) \),\(^10\) and the index for aggregate output, one can rewrite Eq. (16) as follows.

\[
Y_t = K \tilde{Y}_t \left( 1 - \alpha \right) \tilde{Q}_t \tag{17}
\]

Log-linearizing Eq. (17) and omitting the irrelevant constant, yields:

\[
y_t = Y_t + \frac{\sigma}{\sigma} s_t \tag{18}
\]

where \( \sigma = 1 + \alpha(2-\alpha)(\sigma n - 1) > 0 \). According to Eq. (18), the aggregate demand differential between the two countries depends on the terms of trade and, by Eq. (13), the real exchange rate. Observe that \( \sigma n \) plays a key role on that differential. It is assumed that \( \sigma n > 1 \), what seems to be reasonable empirically.

Combining Eqs. (11), (13), (18) and the equilibrium condition for the foreign country \( \left( \gamma^*_Y = c_t^* \right) \), one gets the relationship:

\[
c_t = \tilde{\delta}_t Y_t + (1-\tilde{\delta}) c^*_t \tag{19}
\]

where \( \tilde{\delta} = \frac{1-\alpha}{\alpha} \). Notice that \( \tilde{\delta} \in (0,1) \) for all \( \sigma n > 1 \).

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\(^4\) In Svensson (2000), the UIP also is an additional equilibrium condition. See Backus, Foresti, and Telmer (2001) and Gourinchas and Tornell (2002) for theoretical attempts to model the risk premia.


\(^6\) This condition was derived by Chari, Kehoe and McGrattan (2002) and used by Gali and Monacelli (2000, 2005), among others.

\(^7\) In fact, because Eq. (11) implies a high correlation between real exchange rate and relative consumption, while in the data the correlation is null, this result is usually referred to as consumption-real exchange rate anomaly [see Backus and Smith (1993) and Chari, Kehoe, and McGrattan (2002)].

\(^8\) Monacelli (2005) also uses this assumption to characterize the small open economy.

\(^9\) Notice that using the aggregate labor demand, \( N_t = \sum_q N_t(j) q_t \) and variations in the logarithm of the integral term around the steady state equilibrium are from second order.

\(^10\) Gali and Monacelli (2000, 2005) present a detailed discussion on this condition.
The log-linear optimality condition for the households’ labor supply reads as:

\[ w_t-p_t = \gamma_n \pi_t + \alpha \sigma \theta_t \tag{20} \]

Plugging the price index version of Eq. (12) and Eqs. (19) and (20) into Eq. (15), the firm’s real marginal cost can be rewritten as:

\[ m_t = (\gamma_n + \alpha \sigma) u_t + \sigma (1-\delta) y_t + \sigma (1-\delta) q_t - (1 + \gamma_n) a_t \tag{21} \]

As pointed out before, the monetary authority might use the monetary policy to improve the small open economy terms of trade. This is captured by the real version of the UIP, Eq. (8), which links real exchange rate and the monetary policy instrument. According to Eq. (21), the firm’s real marginal cost is positively related to all arguments except technological shock. Positive shocks to aggregate demand, either domestic or foreign, raise consumption of the home-produced goods, and so affect labor demand and real wages. The real exchange rate has a direct effect on the real marginal cost because real devaluations increase the price of imported final goods, which has a positive impact on the CPI. Given the workers’ wage setting behavior, wage increases and so does the real marginal cost of production. On the other hand, a technological shock increases labor productivity and decreases the firm’s demand for labor per unit of output, reducing real marginal cost.11

3. Aggregate supply

Firms are assumed to set price according to the Calvo (1983) framework. Let \( \theta \) be the probability that firm \( j \) does not change its price in period \( t \). This probability is assumed to be independent of the time period elapsed since the firm’s last price adjustment. The firm’s optimal price setting strategy implies the following path for the PPI inflation:

\[ n_{H,t} = \beta E_t n_{H,t+1} + \lambda \tilde{m}_t \tag{22} \]

where \( \lambda = \frac{(1-\theta)(1-\delta)}{1-\sigma} \) and \( \tilde{m}_t = m_t - m \) is the log-deviation of the firm’s real marginal cost from the marginal cost under fully flexible price, when \( \theta=0 \).

Define the output gap as \( \tilde{y}_t = \gamma_t - \tilde{y}_n \), where \( \tilde{y}_n \) is the output under fully flexible prices. Using Eq. (21) and the definition of output gap, one can write the real marginal cost deviation as:

\[ \tilde{m}_t = \varphi \tilde{y}_t + \frac{1}{1-\sigma} \sigma \eta_t \tag{23} \]

where \( \varphi = \gamma_n + \alpha \sigma \).12

Substituting Eq. (23) into Eq. (22), one finds the small open economy aggregate supply (or Phillips) curve:

\[ n_{H,t} = \beta E_t n_{H,t+1} + \Theta \tilde{y}_t + \Lambda \tilde{q}_t \tag{24} \]

where \( \Theta = \lambda \varphi \) and \( \Lambda = \lambda \frac{1}{1-\sigma} \).

It is immediate to see that Eq. (24) differs from a standard closed economy aggregate supply curve in two ways. Firstly, the degree of openness affects the sensitivity of PPI inflation to movements in the output gap. Secondly, the real exchange rate has a direct effect on PPI inflation. This is because the wage inflation, which is caused by real exchange rate devaluations, affects the firm’s real marginal cost and is transmitted to domestic prices.

The ad hoc cost-push domestic inflation, which is included in the closed economy version of Eq. (24) to generate a monetary policy stabilization trade-off between inflation and output gap,13 is represented here by the real exchange rate. It shows up in Eq. (24) due to both the exchange rate channels of transmission and the private agents’ optimizing behavior. Therefore, the model provides a theoretical background for the policy trade-off faced by the monetary authority.

4. Aggregate demand

The computation of the aggregate demand (or IS) curve departs from the log-linearization of the household’s stochastic Euler equation. It can be written as:

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( \eta_t - \pi_t - \gamma \right) \tag{25} \]

where \( \gamma = -\log \beta \).

Substituting Eqs. (12), (13), (18), and (19) into Eq. (25), using the definition of output gap, and manipulating the resulting equation, one gets the small open economy IS curve:

\[ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} \left( \eta_t - \pi_t - \gamma \right) - \frac{\delta}{\sigma (1-\alpha)} \Delta E_t q_{t+1} \tag{26} \]

where \( \omega = \alpha(2-\alpha)(\sigma^2-1) \) and \( \tilde{y}_n = \tilde{y}_n - \gamma \).

Eq. (26) differs from its closed economy counterpart in three ways. The natural interest rate depends on both the degree of openness and changes in the foreign country output. Also, expected changes in the real exchange rate have a direct effect on the small open economy output gap. As pointed out earlier, one should expect that \( \sigma \gamma \gamma_t \), which implies \( \omega \gamma_t \). In this case, expected real exchange rate deviation \( \Delta E_t q_{t+1} \) has a negative impact on the current period output gap, given that it increases imports and decreases exports. If \( \sigma \gamma \gamma_t \), expected changes in the real exchange rate have no effect on the small open economy aggregate demand because the income and substitution effects cancel out.14 But, from Eq. (24), the real exchange rate is still affecting PPI inflation due to its impact on CPI and the resulting wage inflation.

5. Equilibrium

Definition 1. The log-linear rational expectations equilibrium for the small open economy is completely characterized by a set of processes \( \{ n_{H,t}, 1, n_{T,t}, 1, \ldots \} \) that solves the system of Eqs. (24), (26), (12), (10), and (8), with an optimal monetary policy rule for the nominal interest rate \( (i_{t+1}, 1) \) and conditioned on well-behaved processes for the exogenous variables \( \{ \tau_{t+1}, k, t_{t+1}, k, \ldots \} \).

The definition of equilibrium allows one to derive an important result from the previous model. The equilibrium conditions for the small open economy cannot be fully described by the PPI inflation and output gap equations, as suggested by Gali and Monacelli (2000, 2005). In the so-called canonical representation, they argue that the openness of the economy affects only the slopes of aggregate

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11 Notice that, if \( \alpha=0 \), one gets the firm’s real marginal cost for a closed economy derived by Woodford (2003).

12 Notice that in the steady state, with \( P_{a,t}=P_{a} \), one has that the equilibrium log-real exchange rate is zero (\( \tilde{q}=0 \)), which implies that \( \tilde{q}=0 \).


14 See Svensson and Wijnbergen (1989) for a discussion on the consequences of having a dominating effect in a open economy model.
demand and aggregate supply curves. Eqs. (26) and (24), however, make it clear that the real exchange rate has a direct effect on both curves and it is part of the model’s equilibrium conditions.

The equilibrium conditions are not equivalent to the closed economy counterpart due to the exchange rate channels of transmission for the monetary policy. Given the optimal labor supply schedule (7) and the frictionless labor market, the real exchange rate affects wages through the CPI inflation channel. The wage inflation has a positive impact on the firm’s real marginal cost, leading to domestic inflation. The aggregate demand, on its turn, is affected by real exchange rate movements due to the switching demand effect. Changes in the real exchange rate moves aggregate demand towards the cheapest good in the home currency.

The justification for the canonical representation usually relies on the law of one price. Monacelli (2005) allows for incomplete pass-through in import goods prices in the model of Gali and Monacelli (2000, 2005), and shows that the canonical representation no longer holds. Here, however, deviations from the law of one price are captured by the relative version of the purchasing power parity, where the real exchange rate is stationary. Yet, the canonical representation cannot be used to describe the model’s equilibrium conditions.

The next section derives the optimal interest rate rule that closes the model’s equilibrium conditions and shows that the rule is not isomorphic to the closed economy counterpart.

6. Optimal monetary policy

6.1. Welfare objective

In order to achieve policy goals, the monetary authority minimizes the expected value of a utility-based welfare criterion taking the small open economy rational expectations equilibrium conditions as given. The period welfare loss function, expressed in terms of steady state consumption, is approximated by a quadratic representation cannot be used to describe the model’s equilibrium conditions. The period welfare loss function, expressed in terms of steady state consumption, is approximated by a quadratic representation cannot be used to describe the model’s equilibrium conditions.

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The period welfare loss function is derived by Gali and Monacelli (2000, 2005). Benigno (2004) and Benigno and Benigno (2000) also derive an approximation, but for a two country model under the assumption $\eta \approx 1$.

\[ L_t = \sum_{t=0}^{\infty} \beta^t \left( \pi_{t+1}^2 + \gamma_1 (\theta t - \bar{I})^2 \right) \]

where $\gamma_1 > 0$ and $\bar{I}$ is the target interest rate.

The openness of the economy affects the relative weight placed on output gap stabilization. From the definition of $\gamma_1$ in Eq. (27), one can see that the monetary authority should place a higher weight on output gap stabilization compared to the closed economy counterpart, and that the weight is increasing in the degree of openness. This is needed in order to minimize the arbitrary use of the interest rate to improve the terms of trade, policy that has its trade-off in a higher level of unemployment. The output gap weight is increasing in $\alpha$ because the incentives to arbitrary promote gains in the terms of trade rises with the openness of the economy, as shown by Corsetti and Pesenti (2001). In addition, the monetary policy rule should not respond too aggressively to domestic inflation in order to avoid large fluctuations in output, employment, and wage inflation. An equivalent finding has been obtained for a closed economy when both prices and nominal wages are sticky [e.g., Aoki (2001) and Erceg et al. (2000)].

Notice that when $\alpha = 0$, the welfare criterion Eq. (28) collapses to the one derived by Rotemberg and Woodford (1998, 1999) and Woodford (2003) for a closed economy. On the other hand, if $c_\tau = 1$ (log-utility) and $\eta = 1$, the unconstrained welfare objective Eq. (27) becomes identical to the one derived by Gali and Monacelli (2005). In this particular case, the openness of the economy does not affect the social welfare loss. The real exchange rate, however, is still affecting the aggregate supply in the present model due to the wage inflation caused by exchange rate variations.

An important implication of including the non-negative nominal interest rate constraint is that complete inflation stabilization is no longer an optimal policy. Incomplete inflation stabilization reduces the interest rate variability in response to shocks, which is in accordance with the equilibrium requirement of non-negative interest rate. Moreover, the first best Pareto optimal allocation of zero inflation and output gap is no longer a feasible allocation. The reason is found in the aggregate supply equation, where the presence of the real exchange rate makes it impossible for the monetary authority to have zero inflation and zero output gap simultaneously. The real exchange rate generates the policy trade-off faced by the monetary authority in the small open economy.

It is interesting to notice that the policy trade-off is independent of the source of disturbance. The intuition comes from Eq. (23). Any shock, either domestic or foreign, that moves the output gap in one direction requires a response of the nominal interest rate that moves the real exchange rate in the opposite direction. Thus, it is unfeasible for the monetary authority to simultaneously stabilize inflation and output gap in the small open economy.

6.2. Policy rule under commitment

The monetary authority minimizes the welfare loss objective subject to the rational expectations equilibrium conditions. It solves the optimization problem once and for all and credibly commits itself to:

\[ \left( \frac{a \cdot \pi_t}{\beta} \right) = \left( \frac{1}{1-\beta} \right) \sum_{t=0}^{\infty} \beta^t \left( \pi_{t+1}^2 + \gamma_1 (\theta t - \bar{I})^2 \right) \]

The approximation consists of imposing a requirement that the average interest rate be at least $k$ standard deviations above the lower bound, for a sufficiently large $k$ that make the violations infrequent. See Proposition 6.9 in Woodford (2003) for a formal derivation.

As shown in Woodford (2003), the loss function given by Eq. (28) is equivalent to the one derived under the assumption that transaction frictions are not negligible.

In this case, the whole model collapses to the closed economy studied by Woodford (2003).

Monacelli (2005) finds a similar trade-off between stabilizing domestic inflation and stabilizing either output gap or law of one price deviation.
to the optimal policy rule, having no incentives to deviate from it. Given the private agents’ forward-looking behavior, there are gains from commitment to the policy rule. The credible commitment of the central bank is incorporated into the private agents’ expectations formation. This ends up reducing future expected changes in current variables due to exogenous shocks. The solution applies the Lagrangian techniques discussed in Woodford (2003), who shows that such a procedure delivers the important properties of time consistency and timelessness to the monetary policy rule.

The monetary authority problem is to choose a state contingent plan for \( \{ q_{t+k}, \bar{y}_{t+k}, \pi_{t+k}, q_{t+k}, \bar{e}_{t+k}, \bar{k}_{t+k} \} \) to:

\[
\text{Min } E Q_{29} \sum_{k=0}^{\infty} \beta^{k} \pi_{t+k}
\]

Subject to Eqs. (8), (10), (12), (24), (26), and (28)

The solution implies the following nominal interest rate rule:

\[
i_{t} = \frac{1}{1 + \lambda_{t}} \left( \frac{1}{1 + \lambda_{t}} + \frac{\sigma_{\pi}}{\sigma_{\bar{y}}} \right) i_{t-1} + \Gamma_{y} \bar{y}_{t} - \Gamma_{x} \bar{y}_{t-1}
\]

where the coefficients are:

\[
\Gamma_{0} = \frac{1}{\beta} \left( \frac{1}{1 + \lambda_{t}} + \frac{\sigma_{\pi}}{\sigma_{\bar{y}}} \right) \sigma_{y}, \quad \Gamma_{y} = \frac{\sigma_{\bar{y}}}{\sigma_{\bar{y}}} , \quad \text{and } \Gamma_{x} = \frac{\sigma_{\bar{y}}}{\sigma_{\bar{y}}} (1 + \lambda_{t}).
\]

It is clear from Eq. (30) that the monetary authority should not directly respond to exchange rate movements under the optimal policy. However, the degree of openness affects the policy rule coefficients. Despite the absence of the exchange rate itself in Eq. (30), it is not optimal to follow a fully floating exchange rate regime. The response of the monetary policy to exchange rate movements is indirect, through the domestic inflation and output gap. According to Eqs. (24) and (26), changes in the real exchange rate affect those variables and so require a reaction of the policy instrument. In particular, when there is a foreign shock, the domestic interest rate should follow the foreign interest rate in order to avoid oscillations in the real exchange rate and its effects on domestic variables.

The optimal interest rate rule is not isomorphic to the closed economy counterpart, as can be seen by making \( \alpha = 0 \) in Eq. (30). Clarida, Gali and Gertler (2001) have used the canonical representation proposed by Gali and Monacelli (2000) to derive an interest rate rule and argued that it is equivalent to the one obtained by Clarida, Gali and Gertler (1999) for a closed economy. The openness of the economy, in their setup, affects only the slope of the monetary authority’s reaction function.

This is not the case in the present model due to the exchange rate channels of transmission for the monetary policy. The importance of those channels is also reflected in the specific form assumed by the interest rate rule in the small open economy.

6.3. Policy rule under discretion

The optimization problem given by Eq. (29) is solved period-by-period, taking as given the private sector’s expectations of future variables. There is no commitment to a policy rule and the monetary policy may no longer be time consistent, as pointed out by Woodford (2003) for a closed economy.

The resulting nominal interest rate rule is given by:

\[
i_{t} = \frac{1 + \pi_{0}}{\gamma_{0}} q_{t+k} + \Gamma_{y} \bar{y}_{t} - \Gamma_{x} \bar{y}_{t-1}
\]

where \( \Gamma_{y} \) and \( \Gamma_{x} \) are defined exactly as in the previous section.

Compared to the case under commitment, the findings are now mixed. There is still no direct response of the nominal interest rate to exchange rate variations. The discretionary policy rule, however, is isomorphic to the closed economy counterpart, as can be easily verified by making \( \alpha = 0 \) in Eq. (31). The only difference is on the slope of the policy rule, which depends on the degree of openness of the economy. The policy rules are isomorphic because, under discretion, the interest rate responds only to current period variables, and the social loss function depends on PPI inflation as measure of price variability. The real exchange rate, by the real version of the UIP, is a forward-looking variable and it does not affect the policy problem under a discretionary policy. Therefore, the isomorphism is implied by the discretionary feature of the monetary policy, and it holds even when the real exchange rate plays an important role in the transmission of the monetary policy.

7. Conclusion

This paper focused on the design of a monetary policy rule for a small open economy, with special emphasis on the exchange rate channels of transmission for the monetary policy. The findings showed that, once the transmission mechanisms are in place, the real exchange rate affects the firm’s real marginal cost, the aggregate supply, and the aggregate demand. In the model’s equilibrium conditions, one has to specify the real exchange rate dynamics and the optimal monetary policy rule has a specific form, distinct from the closed economy counterpart.

The openness of the economy has a direct effect on the social welfare objective. The relative weight placed on output gap stabilization is higher than its closed economy counterpart and it is increasing in the degree of openness. This finding reflects the welfare cost, in terms of unemployment, of any arbitrary policy that seeks to promote domestic consumption via gains in the terms of trade. In addition, due to the link between output and real exchange rate, an excessive weight placed on domestic inflation stabilization might generate instability in the economic activity. This result finds a parallel in the recent literature on optimal monetary policy for a closed economy when prices and wages are sticky.

The monetary authority faces a trade-off between stabilizing inflation versus output gap. The trade-off, however, is due to the real exchange rate and not to the ad hoc cost-push domestic inflation shock that is appended to the closed economy aggregate supply. The real exchange rate captures the wage inflation that is a component of the firm’s real marginal cost. Any shock that requires an increase in the nominal interest rate leads, through the exchange rate, to movements in the firm’s real marginal cost in the opposite direction. As a result, the monetary authority is not able to stabilize inflation and output gap simultaneously.

From the monetary authority’s optimization problem, the implied policy regime is a domestic inflation target coupled with a dirty floating of the exchange rate. In the reaction function, the nominal interest rate responds to domestic inflation and output gap. Those variables, however, are directly affected by the real exchange rate
References


